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Homotopy Quantum Field Theory

With Appendices by Michael Müger and Alexis Virelizier

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Homotopy Quantum Field Theory (HQFT) is a branch of Topological Quantum Field Theory founded by E. Witten and M. Atiyah. It applies ideas from theoretical physics to study principal bundles over manifolds and, more generally, homotopy classes of maps from manifolds to a fixed target space.

This book is the first systematic exposition of Homotopy Quantum Field Theory. It starts with a formal definition of an HQFT and provides examples of HQFTs in all dimensions. The main body of the text is focused on 2-dimensional and 3-dimensional HQFTs. A study of these HQFTs leads to new algebraic objects: crossed Frobenius group-algebras, crossed ribbon group-categories, and Hopf group-coalgebras. These notions and their connections with HQFTs are discussed in detail. The text ends with several appendices including an outline of recent developments and a list of open problems. Three appendices by M. Müger and A. Virelizier summarize their work in this area.

The book is addressed to mathematicians, theoretical physicists, and graduate students interested in topological aspects of quantum field theory. The exposition is self-contained and well suited for a one-semester graduate course. Prerequisites include only basics of algebra and topology.

Contents:

Generalities on HQFTs · Basic definitions · Cohomological HQFTs and transfer · Aspherical targets · Hermitian and unitary HQFTs · Proof of Lemmas 1.3.1-1.3.3

Group-algebras \cdot *G*-algebras \cdot Inner products and Frobenius *G*-algebras \cdot Crossed Frobenius *G*-algebras \cdot Transfer \cdot Semisimple crossed *G*-algebras \cdot Semisimple crossed Frobenius *G*-algebras \cdot Hermitian *G*-algebras

Two-dimensional HQFTs \cdot The underlying *G*-algebra \cdot Computation for cohomological HQFTs \cdot Equivalence of categories \cdot Proof of Theorem 3.1 \cdot Hermitian two-dimensional HQFTs

Biangular algebras and lattice HQFTs \cdot Frobenius *G*-algebras re-examined \cdot Biangular *G*-algebras \cdot Lattice HQFTs \cdot Skeletons of surfaces \cdot Hermitian biangular *G*-algebras

Enumeration problems in dimension two · Enumeration problem for homomorphisms · Linear representations of Γ and cohomology · Projective representations of Γ · Properties of κ_{ρ} and ζ_{ρ} · Equivalence of two approaches · A generalization and a proof of Theorem 1.2.1 · A homological obstruction to lifting · Applications of Theorem 1.2.1 · Further applications of Theorem 1.2.1

Crossed *G*-categories and invariants of links \cdot *G*-categories \cdot Crossed, braided, and ribbon *G*-categories \cdot Colored *G*-tangles and their invariants \cdot Colored *G*-graphs and their invariants \cdot Trace, dimension, and algebra of colors

Modular *G*-categories and HQFTs \cdot Modular crossed *G*-categories \cdot Invariants of 3-dimensional *G*-manifolds \cdot Homotopy modular functor \cdot Two-dimensional HQFT \cdot Three-dimensional HQFT

Miscellaneous algebra \cdot Hopf *G*-coalgebras \cdot Canonical extensions \cdot Transfer of categories \cdot Quasiabelian cohomology of groups \cdot Remarks on group-algebras

Appendix 1: Relative HQFTs

Appendix 2: State sum invariants of 3-dimensional *G*-manifolds

Appendix 3: Recent work on HQFTs

Appendix 4: Open problems

Appendix 5: On the structure of braided crossed G-categories by M. Müger

Appendix 6: Algebraic properties of Hopf G-coalgebras by A. Virelizier

Appendix 7: Invariants of 3-dimensional G-manifolds from Hopf coalgebras by A. Virelizier

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