Master class 2016-2017 in geometry, topology and physics

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Introduction to Quantum Topology I

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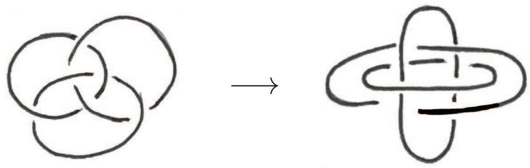
Exercise sheet 5

Exercise 1.
Find a sequence of Reidemeister moves transforming the first link diagram into the second one:



Exercise 2.
Compute the Jones polynomial of the figure-eight knot below in two ways: first using the Kauffman bracket definition and then using the skein relation.

Exercise 3.
Let L be an oriented link with m components. Prove that

$$V_L(1) = (-2)^{m-1}.$$

Exercise 4.
Let L be an oriented link. The *mirror image* L^* of L is the link obtained by changing every crossing of L to its opposite while preserving the orientations of the components. Prove that

$$V_{L^*}(t) = V_L(t^{-1}).$$

Exercise 5.
Consider an arbitrary connected sum $L_1 \# L_2$ of two oriented links L_1 and L_2 obtained by connecting a component of L_1 with a component of L_2. Prove that

$$V_{L_1 \# L_2}(t) = V_{L_1}(t)\, V_{L_2}(t).$$

Exercise 6.
Let $L_1 \sqcup L_2$ be the disjoint union of two oriented links L_1 and L_2. Prove that

$$V_{L_1 \sqcup L_2}(t) = (-t^{\frac{1}{2}} - t^{-\frac{1}{2}})\, V_{L_1}(t)\, V_{L_2}(t).$$

Exercise 7.
Prove that the Jones polynomials of the following two non-isotopic links coincide.