

INTRODUCTION TO QUANTUM TOPOLOGY I

EXERCISE SHEET 6

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**Exercise 1.**

Are the left-handed and right-handed trefoils isotopic?

**Exercise 2.**

Does the Jones polynomial of a knot depend on its orientation? What about links?

**Exercise 3.**

The HOMFLY polynomial of an oriented link  $L$  is a 2-variable Laurent polynomial  $H_L(t, z)$  which is invariant under isotopies, satisfies the skein relation

$$t^{-1}H_{L_+}(t, z) - tH_{L_-}(t, z) = zH_{L_0}(t, z)$$

and  $H_U(t, z) = 1$  for the unknot  $U$ . Verify that the HOMFLY polynomial generalizes the Jones polynomial. Compute the HOMFLY polynomial of the trefoil and the figure-eight knot.

**Exercise 4.**

A Seifert surface of an oriented link is a compact connected oriented surface embedded in  $\mathbb{R}^3$  whose boundary is the link.

**a.** Prove that every oriented link has a Seifert surface.

*Hint:* smooth the crossings of a connected diagram of the link to obtain disjoint loops which bounds disks that are then connected by bands.

**b.** Find Seifert surfaces of the trefoil and the figure-eight knot.